12/8/17 "After the game, the king and the pawn go into the same box." -Italian Proverb

HW: "Remainder and Factor Theorem" worksheet Exercise Set A #1a-d
Test 2 on Tuesday 12/19

AIM: What are the Remainder and Factor Theorems?

Warm Up:
On Handout

$$Let\ P(x) = 3x^5 + 5x^4 - 4x^3 + 7x + 3$$

(a) Find the quotient and remainder when P(x) is divided by x+2

$$3x^4 - 1x^3 - 2x^2 + 4x - 1 + \frac{5}{x+2}$$

$$x+2 \overline{)\,3x^5 + 5x^4 - 4x^3 + 0x^2 + 7x + 3}$$
$$-(3x^5 + 6x^4)$$
$$-1x^4 - 4x^3$$
$$-(-1x^4 - 2x^3)$$
$$-2x^3 + 0x^2$$
$$-(-2x^3 - 4x^2)$$
$$4x^2 + 7x$$
$$-(4x^2 + 8x)$$
$$-1x + 3$$
$$-(-1x - 2)$$
$$5$$

$$\frac{3x^5}{x} = 3x^4$$

$$\frac{-2x^3}{x} = -2x^2$$

$$\frac{4x^2}{x} = 4x$$

(b) Find P(-2) plug -2 in for x in P(x)

$$P(-2) = 3(-2)^5 + 5(-2)^4 - 4(-2)^3 + 7(-2) + 3$$

$$P(-2) = 5$$

Remainder Theorem:

If the polynomial $P(x)$ is divided by $x - c$, then the remainder is the value $P(c)$.

1) Set the divisor = 0 and solve for x.

2) Plug that value into the Polynomial to get the remainder

1. Let $P(x) = x^3 - 2x^2 + 3x - 1$. Find $P(3)$ using 2 different methods.

Method 1

$P(3) = 3^3 - 2(3)^2 + 3(3) - 1$

$P(3) = 17$

That is the value we get when we set the divisor = 0

$X = 3$ comes from $(X - 3)$

Method 2

$$
\begin{array}{r|l}
 & x^2 + x + 6 \\
\hline
x - 3 & \cancel{x^3} - 2x^2 + 3x - 1 \\
 & -(\cancel{x^3} - 3x^2) \\
\hline
 & \cancel{x^2} + 3x \\
 & -(\cancel{x^2} - 3x) \\
\hline
 & 6x - 1 \\
 & -(6x - 18) \\
\hline
 & 17
\end{array}
$$

Factor Theorem:

A polynomial $P(x)$ has a factor of $x-c$ if and only if $P(c)=0$.

* If there is no remainder then the divisor is a factor. If we plug in the value we get when setting the divison = 0 into the polynomial and get 0 then its a factor.

2. Show that $x-2$ is a factor of $P(x)=x^3-3x^2+7x-10$.

$x-2=0$

$x=2$

$P(2)=2^3-3(2)^2+7(2)-10$

$P(2)=0$

Therefore $x-2$ is a factor of $P(x)$

3. (a) Use the factor theorem to show that $x+3$ is a factor of $P(x)=x^3-x^2-8x+12$

(b) Factor $P(x)$ completely.

4. Let $P(x) = x^3 - 7x + 6$.

(a) Show that $P(1) = 0$

(b) Factor $P(x)$ completely.

5. Find a polynomial of degree 4 that has zeros $-3, 0, 1,$ and 5.